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A COSMOLOGICAL MODEL FOR OUR UNIVERSE. I.

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ABSTRACT

In this paper we attempt to construct a cosmological model for our Universe, based on currently available astronomical observations and two basic assumptions: (1) Einstein's theory of general relativity is strictly valid without the cosmological constant (2) the existing laws of physics are strictly valid. We have disregarded the Mach's principle because its meaning is ambiguous.

First we review the present status of knowledge of astronomical observations of our Universe. Next we review the cosmological models acceptable under the two assumptions cited above. We then formulate our method of construction of a cosmological model. Using statistical physics for equilibrium and non-equilibrium processes, we reconstruct the course of evolution of our Universe. From the 3 ^{o}K cosmic background radiation, the upper limit of neutral hydrogen density in intergalactic space, and the density of matter due to galaxies, we have obtained an upper limit for the allowable matter-density of the Universe. We have found that the bulk part of matter-density in our Universe is in galaxies.

We also calculate the evolution of antimatter in our Universe and we have found that antimatter must be totally

absent from our Universe. Hence there are no antiworlds, antigalaxies, etc.

Based on our result we have obtained a value of 0.02 for the deceleration parameter q_o . This is to be compared with the value of 0.5 obtained recently by Sandage. In view of the uncertainties associated with both values, these two values must be regarded to be consistent with each other. Consequently no suggestion of violation of any physical law is made.

In conclusion, our Universe can be described by an open cosmological model of the Friedman type.

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I. Introduction.

The desire to know the past history as well as the future course of evolution of the Universe has long resided in Man's mind, but the degree of his success in his search for this knowledge is limited by his ability to observe the Universe, not by his power of comprehension. In recent years tremendous progresses have been rendered to astronomical research by the use of modern techniques of observation and data processing, and by extending the wave length region of observation to nearly all parts of the electro-magnetic spectrum. Despite all these technological advances, Man is still limited by his ability to sample the properties of the matter in different parts of the Universe.

Man often supplements his ignorance by his imagination, and whenever any information is missing, he makes postulates about the missing link. The nature of the postulates he makes often reflects his knowledge of the system, and hence, they are posted according to his past experiences. In cases where only a small link of information is missing, Man's rich imagination has guided him to enlarge his horizon and to make new discoveries. In cases where a large portion of information is lacking, these postulates are often invented according to the inventor's personal conviction.

In this paper we shall take up the view that if we abandon the idea that man must invent some principles for the Universe to abide with, we can perhaps enlarge our knowledge without having a biased opinion <u>ab initio</u>. Therefore we shall not postulate any cosmological principle, nor invent new laws of physics. Out of all phenomena in the Universe we can only make a limited number of observations with a limited degree of precision. It is not clear if we will ever possess a complete knowledge of the Universe. We therefore have to regard the Universe as an open system. This means that there might be information which is forever inaccessible to us, and we may never be able to answer all questions in cosmological problems. Under this view, clearly the cosmological model we shall arrive at will not be unique, since we shall base our work on existing laws of physics. This cosmological model will be constructed, however, on the basis of observation. Since new observations are constantly being made, our cosmological model will have to be constantly modified.

In this paper we shall assume:

(i) That the general relativity theory as formulated by Einstein without the cosmological constant is valid.

(ii) That the currently accepted physical laws are strictly valid in space and time.

Although thereare a number of modified versions of the general relativity theory, in their predictions of observations they either do not differ from Einstein's theory or the difference is beyond the current limits of experimental accuracy. Einstein's theory has the beauty of being the simplest among all, and it contains only one constant to be determined from the classical limit; that constant is the gravitational constant. Its success in predicting the advancement of the perihelion of the orbit of the planet Mercury is an impressive, if not conclusive evidence of its validity.(1)

We shall adopt the point of view that, unless there is experimental evidence that a physical law is clearly violated, or that a physical constant is a function of time, we shall assume the converse. That is, we shall assume that all physical laws are strictly valid and that all physical constants, including the gravitational constant, are constant in space and time. We do this because our interest is to find a relation among currently accepted physical laws, astronomical observations, and a given cosmological model. Out of the infinite ways that a violation may occur, the chance for an arbitrary postulate to be correct is small.

In addition, we shall avoid the application of Mach's principle to cosmological models. Theoretically, Mach's principle provides an operational definition of an inertial system and of a mass in terms of the distribution of other masses in the Universe. It has been shown that Mach's principle cannot be unambiguously applied to open cosmological models, in which the expansion of the Universe never ceases.[2] It has been argued that the inapplicability of Mach's principle to an open cosmological model is the reason for not accepting an open cosmological model. However, Mach's principle has not been formulated in an unambiguous way. Further, if an open universe is embedded in a closed universe of a much larger dimension and a much lower matter-energy density, Mach's principle can still be satisfied with respect to the closed Universe without referring to the structure of the smaller, open Universe. We therefore feel that a premature inclusion of Mach's principle in theory will not help clarify existing problems in cosmology, but will tend to include unnecessary complications. Henceforth, we shall avoid the mention and use of Mach's principle.

The plan for the rest of this paper is as follows: In Section II we shall discuss the validity of certain physics laws in terms of some very precise experiments. All physical laws are interrelated by theory and this interrelationship is used to interpret the theoretical implications of precise experiments. Unfortunately, the number of precise experiments is too small to make the use of theoretical interpretation as a basis for a rigorous argument. Much of the explanation will therefore depend on the validity of some theoretical principles. Hopefully in the future this situation will improve, when more such precise experiments available.

In Section III we shall summarize the available astronomical observations relevant to cosmology. In Section IV we shall summarize the available cosmological models, including some which use modified laws of physics.

In Section V we shall discuss the classical tests of the cosmological theory. In Section VI one of the most important properties of a cosmological model, the question of closure will be treated. We shall propose a method to determine the closure property of the Universe based on the available data on the intergalactic neutral hydrogen density and the density due to galaxies, and based on the use of nonequilibrium thermodynamics. In Sections VII-IX we shall summarize the theory of equilibrium and nonequilibrium statistical thermodynamics, based on two body interaction processes. These theories will be used to develope a cosmological model for our Universe.

In Sections X and XI the problem of annihilation of particle pairs in an evolving cosmological model will be formulated and a solution obtained for the case of a radiation-filled universe. It is believed that if our Universe originated from a singular origin, then radiation is dominant in early epochs. It will be shown that our Universe cannot be composed of an equal population of nucleons and antinucleons.

In Section XII a cosmological model for our Universe is developed, based on previous sections. We shall apply the solution obtained in Section X to estimate the ionized to neutral hydrogen ratio, to the neutrino energy densities of the Universe, and to the overall matter energy density of the Universe. It is tentatively concluded, that galaxies comprise the bulk of the matter and energy in the Universe. Consequently, it is unlikely that our Universe can be described by a closed model.

II. Precise Experiments.

Recent speculations propose that certain physics laws may be only approximate in nature and that a small degree of violation of these physical laws may have important cosmological consequences. In the history of physics, new physical laws are often discovered as a result of the failure of old laws to interpret natural phenomena; but such a discovery necessarily introduces radical departures from the older theory and, as is always the case, the older laws become a special case of the newly established theory. It is difficult to imagine that a violation will take place in a fragmentary manner, i.e., the gross structure of old laws remains valid while a small violation takes place here and there. Such violations appear to be too arbitrarily imposed. Hence, we shall not advocate the invention of a new law by violating an old law in a fragmentary manner.

In recent years, a number of highly accurate experiments of fundamental importance have been performed, which have excluded some proposed "violations laws". Since these laws are of fundamental importance, they are discussed briefly below:

(i) <u>Isotropy of Inertial Mass</u>. It was proposed by Salpeter and Coconni[(3)] that in accordance with Mach's principle, the inertial properties of matter may depend on the presence of nearby masses and that this dependence may induce an anisotropy in the inertial properties of matter. Dicke has shown, however, that even if such anisotropy existed, it would have been unobservable, according to the theory of general relativity.[(4)] Nevertheless, an experiment performed under the direction of V. W. Hughes showed that to one part in 10^{22},[(5)] there is no detectable anisotropy of matter. Dicke argued that this experiment thus supports the principle of covariance, if Salpeter and Coconni's original postulate is correct that in accordance with Mach's principle the inertial properties of matter depends on the local distribution of other masses.

(ii) <u>The Equality of the Electric Charge of an Electron and a Proton</u>. Lyttleton and Bondi[(6)] have suggested that a small departure of equality of the electric charge of an electron and a proton can cause the Universe to expand. If the Universe is taken to be on the whole electrically neutral, then the charge inequality hypothesis will only imply an excessive number of electrons or protons. Hence, the Universe must be taken to be composed of an equal number of electrons and protons and the small charge difference can cause the Universe to expand. Why the total number of electrons and protons must be such that there is a net charge is not explained. The presence of a small charge density everywhere in the Universe will invalidate gauge invariance, and Coloumb's law. The degree of violation as proposed by Bondi and Lyttleton is one part in 10^{19}. An experiment performed by V. W. Hughes et al[7] indicated that the atomic argon is charge neutral to one part in 10^{21}, indicating that no such inequality of charge exists.

(iii) Lorentz Invariance of the Electric Charge. Hughes' experiment is of great theoretical importance because it gives a firm experimental confirmation of some consequences of Lorentz invariance.[8] In an atom the average square velocity of an electron is not zero and in the case of argon it amounts to 0.01 c^2. The electric charge is a Lorentz invariant. However, it may be assumed that the charge is dependent on the velocity of the electron, the dependence can be written as

$$q = q_o \left(1 + a \frac{v^2}{c^2}\right) , \tag{1}$$

where q is the charge of a moving electron of velocity v, and q_o

is the charge at rest. Then, from the accuracy of Hughes' experiment, the value of the constant "a" is less than 10^{-17}.

(iv) The Equivalence of Mass and Energy. Eötvös-Dicke Experiment. Newtonian theory of gravitation implies that the gravitational mass (the property of an object in response to gravitation) is equivalent to the inertial mass. In Einstein's theory it is further assumed that mass is equivalent to energy and the gravitational mass is exactly equivalent to the inertia mass. Eötvös[(9)], and recently, Dicke[(10)], have shown that to a very high degree of accuracy, the equivalence principle is valid. The upper limit of Eötvös' classical experiment is around a few parts in 10^{8} and in Dicke's experiment the accuracy is around five parts in 10^{12}. One of the other consequences of the equivalence principle, is the red shift of an electromagnetic radiation in a gravitational field. Using the Mössbauer effect, Pound has shown that the red-shift relation has been shown to be valid to a few parts in one thousand.[(11)]

(v) The Longevity of Protons. (Baryon number conservation law).

If the baryon number is not conserved, then a proton can decay into π-mesons. In an experiment, Reines has established that the experimental lower limit of the natural lifetime of a proton against decay into π mesons is 10^{26} years.[(12)] As a comparison, the age of the Universe is 10^{10} years.

From these experiments we see that there is a unity in the structure of physical laws, and often the validity of these laws is established to such a degree that even if a violation may take place, it will not have any cosmological effect at all (e.g., the charge inequality). Hence, it appears prudent to exhaust all possibilities of existing laws of physics before venturing into inventing new physics laws.

III. Astronomical Information Concerning the Universe.

Because of our immobility in the Universe, quantities which are generally regarded as definable in terms of direct measurements in the laboratory become definable only when additional assumptions are made. So far, electromagnetic radiation emitted by galaxies and quasars provides the only means of detection of extragalactic matter.

In cosmological theories the most important information is the state of motion of matter and the distribution of matter and other forms of energy. It is important to know the distance of galaxies and other forms of matter, the mass, and the velocity.

(i) Distance determination. In laboratory, direct comparison or triangulation provides the two usual means of measurement of distance. Direct measurement of distances can also be made by sending radar pulses or probes. The longest distance ever measured directly by radar pulses is the distance between the sun and the earth (1.5×10^{13} cm), and that distance from the Earth to Mars by sending the interplanetary probe Mariner IV. The distance is 2×10^{13} cm. Using methods of triangulation and the diameter of the earth's orbit around the sun as the base line, direct measurement of stellar distances can be extended to 100 light years, or 10^{21} cm. The orbital motion of binary stars, assuming the validity of Kepler's law, provides us information distances of the order of 10^5 light years. The period-luminosity relation of Cepheid variables may be used to extend the measurable distance further to 10^7 light years. But there are intrinsic variations in the period-luminosity relation in Cepheids. It therefore involves a greater risk to apply the period-luminosity relation tc other galaxies. If the intrinsic brightness of the brightest star in galaxies is assumed to be uniform, then the apparent magnitude of the brightest stars can also be used to determine the distance to a galaxy.

Beyond a distance of 10^7 light years, it is no longer possible to resolve individual stars in a galaxy, and the distance can only be obtained from the apparent magnitude of a galaxy, using a mean magnitude versus distance relation. Because of intrinsic variations in the absolute magnitude of galaxies, this method does not give a reliable answer if it is applied to a single galaxy. However, a more consistent answer is obtained when applying it to a cluster of galaxies. This method of distance determination is closely related to the red-shift of distant galaxies which will be discussed below.

The apparent bolometric magnitude m_b is related to the distance r by the inverse square law

$$m_b = 5 \log r + C \quad , \tag{2}$$

where C is a constant, provided that (a) the absolute magnitude of sample galaxies are the same and (b) the curvature of space-time is small up to a distance r . Problems involved in using Eq. (2) will be discussed later.

(ii) Dynamical motions of galaxies in the Universe. Early in this century it was discovered that some extragalactic nebulae were receding from us a speeds up to 1000 km/sec, as indicated by the red shift H and K absorption lines. Using the brightest stars in a galaxy as distance indicators, Hubble found that the recession velocity is correlated to the distance by the following equation:

$$V = \frac{\Delta\lambda}{\lambda} c \;=\; H_o r \tag{3}$$

where H_o is the Hubble constant.[13] At present the value of H_o is between 7.5 to 10 cm/sec-pc. Using the distance-magnitude relation given by Eq.(2), it is found that the recession velocity of distant galaxies also obey Eq. (3) approximately. There is, however, a large spread of data presumed to be primarily the result of variation of the intrinsic brightness of individual galaxies.

Sandage plotted the apparent magnitude of the brightest members of clusters of galaxies versus the red shift $\frac{\Delta\lambda}{\lambda}$.[14] He obtained a remarkably well-defined linear relation between the apparent magnitude and the red shift, up to $\frac{\Delta\lambda}{\lambda} \approx 0.1$, corresponding to a velocity of 0.1 C.

The distance corresponding to this velocity (Eq. 3) is around 2×10^9 light years. In all theories and cosmological and relativistic to the linear velocity red shift relation correction is of the order of $(\frac{\Delta\lambda}{\lambda})^2$. Hence, we can neglect both corrections up to a distance of 2×10^9 light years. This linear relation between the apparent magnitude and the red shift can be interpreted as the validity of the assumptions: (a) of the inverse square law; (b) that there is only a small variation in the intrinsic brightness of the brightest members of clusters of galaxies.

In obtaining the red-shift-magnitude relation in Fig. 1, several corrections to new observational data have to be made. One of the most important correction terms is the K-term, whose nature is as follows: Because the observable part of the optical spectrum is limited to 3500 - 10000 Å, owing to red shift the observed spectra of distant galaxies corresponds to the unobserved part of nearby galaxies. If Eq. (2) is applied to the visual magnitude (which measures the average energy flux at $\lambda = 5500$ Å), then a correction term usually referred to as the K-term must be added. In the absence of a better knowledge of the spectrum of galaxies, this correction term depends strongly on theoretical calculations.

At distances greater than 2×10^9 light years, the light from distant galaxies is so faint that it may not be distinguished from the night-emission of the earth's atmosphere. It is difficult, if not impossible, to obtain accurate information on these distant galaxies.

(iii) Distribution of galaxies in the Universe. At present there is a relatively good data on the distribution of galaxies up to a distance of 10^9 light years. It is customary to define the quantity $N(m)$, which is the number of galaxies per square degree brighter than a magnitude m. Using Eq. (2), N(m) then becomes the number of galaxies enclosed up to a distance r, and $\frac{dN(m)}{4\pi r^2 dr}$ is the number density of galaxies at r. Up to a distance of 10^9 light years, the observed $N(m)$ agrees with that given by a Eucleidean space with uniformly distributed galaxies.[15] The derived matter-energy density (due to galaxies) is 7×10^{-31} g/cm^3.[16]

The properties of the Universe up to a distance of 10^9 light years is in agreement with the concept of homogeneity and isotropy.

(iv) Other forms of matter-energy density. In addition to galaxies, the Universe appears to be filled with an isotropic radiation, which, interpreted as a black body radiation, has a temperature of 3° K.[17] At present, only a part of the spectrum is available. The theoretical spectrum and the observed points are shown in Fig. 1. A 3 $^{\circ}$K black body radiation gives an energy density of 6×10^{-34} g/cm^3, which is 10^{-3} times smaller than the density of galaxies.[18]

One form of intergalactic matter is neutral hydrogen. There is no positive information on intergalactic matter. If the red-shift of the quasars is taken to be cosmological, then from the shape of the emission line Hα, Gunn and Peterson[19] and later, Bahcall and Salpeter[20] concluded that the density of intergalactic neutral hydrogen cannot exceed 10^{-34} g/cm^3.

There is little or no information on ionized hydrogen and other forms of matter, or energy. We shall discuss these problems in Section XI.

IV. Cosmological Models.

(i) Conventional cosmological models.

Astronomical information indicates that up to a distance of 2×10^9 light years, the distribution of galaxies shows homogeneity and isotropy. If isotropy and homogeneity are assumed to be properties of the Universe, and if we exclude static models, then it can be shown[20] that the most general form of the line element ds in a co-moving system with respect to matter is given by*

$$ds^2 = - e^{f(r)+g(t)} (dr^2 + r^2 \, d\theta^2 + r^2 \sin^2\theta \, d\varphi^2) + dt^2 , \qquad (4)$$

where θ, φ are the polar angles, r is the radial coordinate and t is the proper time. We have used Tolman's notation.[21] f(r) and g(t) are functions of r and t, which are determined from field equations. Using Eq. (4), Einstein's field equations become

$$8\pi \, T_1^1 = - e^{(f(r)+g(t))} \left(\frac{f'^2}{4} + \frac{f''}{r}\right) + \ddot{g} + \frac{3}{4} \dot{g}^2$$

$$8\pi \, T_2^2 = 8\pi \, T_3^3 = - e^{-(f(r)+g(t))} \left(\frac{f''}{2} + \frac{f'}{2r}\right) + \ddot{g} + \frac{3}{4} \dot{g}^2 \qquad (5)$$

$$8\pi \, T_4^4 = - e^{-(f(r)+g(t))} \left(f'' + \frac{f'^2}{4} + \frac{2f'}{4}\right) + \frac{3}{4} \dot{g}^2 \quad .$$

A prime and a dot refer to differentiation with respect to r and t

*Here we adopt the system of units such that c=G=1, where G is the gravitational constant and c is the velocity of light. In this system of units, the unit of length is cm the unit of time is $\frac{1}{c} = 3.335 \times 10^{-11}$ sec, and the unit of mass is $M = 1.349 \times 10^{28}$ g.

respectively, and T^i_j is the stress energy tensor. The cosmological constant Λ has been put to zero, according to the current practice. If space isotropy is assumed, then $T^1_1 = T^2_2 = T^3_3$, and a first integral can be obtained by equating the first and the second equations in Eq. (5):

$$e^{f(r)} = \frac{A}{\left(1+ \frac{r^2}{4R_o^2}\right)} \tag{6}$$

where R_o^2 is a constant which can be positive, negative, or zero. Absorbing the constant A to the undetermined function $e^{g(t)}$, we can write the most general form of the line element as

$$ds^2 = - \frac{e^{g(t)}}{1+ \frac{r^2}{4R_o^2}} \; (dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2) + dt^2 \; . \tag{7}$$

Since

$$T^1_1 = T^2_2 = T^3_3 = - P \; , \quad T^4_4 = \rho \quad , \tag{8}$$

where P is the pressure and ρ is the matter-energy density, by using Eqs. (7) and (8), the field equations (5) become

$$8\pi \; P = - \frac{1}{R_o^2} \; e^{-g(t)} - \ddot{g} - \frac{3}{4} \dot{g}^2 \tag{9}$$

$$8\pi\ \rho = \frac{3}{R_o^{\ 2}}\ e^{-g(t)} + \frac{3}{4}\ \dot{g}^2\ . \tag{10}$$

$e^{\frac{1}{2}g(t)}$ is the metric for length. We can therefore write

$$e^{\frac{1}{2}g(t)} = \frac{\ell}{\ell_o}\ , \tag{11}$$

where ℓ is the proper length at time t and ℓ_o is a constant. P and ρ are all functions of ℓ, and hence are implicit functions of time t .

Substituting Eq. (11) into (10), we obtain

$$\frac{d\ell}{dt} = + \left[\ \frac{8\pi}{3}\ \rho\ell^2 - \left(\frac{\ell_o}{R_o}\right)^2\ \right]^{\frac{1}{2}} \tag{12}$$

where we have chosen the + sign for the square root to fit the expansion of the Universe.

As we shall show later for radiation $\rho \propto \ell^{-4}$, and for matter $\rho \propto \ell^{-3}$, hence at an earlier epoch, other quantities in the parenthesis on the right hand side of Eq. (12) can be neglected against $\rho\ \ell^2$. We then find that

$$\frac{d\ell}{dt} = \left(\frac{8\pi}{3}\ \rho\ \ell^2\right)^{\frac{1}{2}} \tag{13}$$

which gives the solution $\ell \propto t^{\frac{1}{2}}$. This means that there is a time in the past when the radius of the Universe is zero. Thus, general relativity theory predicts a definite origin of the Universe.

From Eq. (13), we find that in an early epoch, for a radiation filled Universe,

$$H_o(t) = \frac{d \ln \ell}{d t} = \frac{1}{2t} , \quad T = T' t^{-\frac{1}{2}} , \tag{14}$$

and for a matter filled universe with a negligible pressure,

$$H_o(t) = \frac{2}{3t} , \qquad T = T'' t^{-2/3} \tag{15}$$

where T' and T'' are constants.

To see whether the expansion will ever stop, the field equations and the metric will be written in another form originated by Robertson.[22] If we make the substitution that

$$r'^2 = \frac{r^2}{|R_o^2|} , \qquad R_o^2 = k |R_o^2| , \tag{16}$$

(note that R_o^2 may be positive or negative, so that the value of k is +1, -1, or 0), then Eq. (7) becomes

$$ds^2 = dt^2 - \frac{|R_o^2| e^{g(t)}}{(1+ k \frac{r'^2}{4})} (dr'^2 + r'^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2). \tag{17}$$

Let $R(t) = |R_o{}^2|\ e^{\frac{1}{2}g(t)}$. (R(t) is the scalar curvature of the Universe.) Then the field equations (9) and (10) become

$$\frac{\dot{R}^2}{R^2} + \frac{2\ddot{R}}{R} + 8\pi P = -\frac{k}{R^2}$$
$$\frac{\dot{R}^2}{R} - \frac{8\pi\rho}{3} = -\frac{k}{R^2} \quad . \qquad (18)$$

A positive value of k corresponds to a closed Universe, a negative value of k to an open Universe, and a null value of k to an Eucleidean Universe. The field equations will now be rewritten in a manner so as to correlate the curvature of the Universe to the overall matter-energy density of the Universe.

Clearly the Hubble constant can be written in analogy to Eq. (11):

$$H_o = \frac{\dot{R}}{R} = \frac{d\ell nR}{dt} \quad . \qquad (19)$$

We now define a deceleration parameter q_o :

$$q_o = -\frac{\ddot{R}}{R\ H_o{}^2} = -\frac{d\ell n\dot{R}}{dt} \Big/ \frac{d\ell nR}{dt} \quad . \qquad (20)$$

Equating the two equations of (18), we find

$$\frac{\ddot{R}}{R} + 4\pi\left(\frac{\rho}{3} + P\right) = 0 . \tag{21}$$

Using Eqs. (19) and (20) to simplify Eq. (21), we find

$$3 P + \rho = \frac{3 H_o^2 q_o}{4} \tag{22}$$

or in c.g.s. units,

$$\frac{3 P}{c^2} + \rho = \frac{3 H_o^2 q_o}{4 G} = 3.21 \times 10^{-29} q_o \left(\frac{H_o}{100 \text{ km/sec/megaparsec}}\right)^2 \tag{22a}$$

This equation relates the matter-energy density $3 P + \rho$ to the deceleration parameter q_o and the Hubble constant H_o.

Substituting Eq. (22) into the first of Eq. (18) and using Eq. (19) and (20) to simplify results, we obtain

$$\frac{k}{R^2} = \frac{4\pi}{3q_o} [\rho(2q_o - 1) - 3 P]$$

$$= \left\{ \frac{4\pi G}{3q_o} [\rho(2q_o - 1) - \frac{3P}{c^2}] \right\}_{\text{in c.g.s. units.}} \tag{23}$$

Eq. (23) relates the nature of the geometry of our Universe to the matter-energy density ρ, the pressure, and the deceleration parameter q_o of the Universe. For a Universe in which matter is dominant, then $P \sim 0$ and the condition for a closed Universe $(k = +1)$ are the same as having

$$q_o > \frac{1}{2} \quad .$$

For a Universe in which radiation is dominant, so that $P = 3 \rho c^2$, the condition for a closed Universe is

$$q_o > 1 \quad .$$

Note that H_o is an observable quantity (obtained directly from the red-shift of distant galaxies). Therefore, a knowledge of the matter-energy density in the Universe will decide the closure properties of the Universe. The more conventional tests of cosmological theories center around measuring properties of very distant galaxies to obtain a value of q_o. We shall adopt the use of the matter-energy density to construct a cosmological model.

(ii) <u>Unconventional Cosmological Models</u>.

(a) Continuous creation or steady state model.

Gold and Bondi first had the idea of this model, but it was finalized by Hoyle into a mathematically rigorous theory.[23] This model has been exploited extensively by Hoyle and his associates. Briefly speaking, this type of model is based on two <u>cosmological</u> <u>principles</u>:

(1) the Universe is homogeneous in space

(2) the Universe is homogeneous in time

As we have seen, (1) is an observed property of the Universe up to a distance of 2×10^9 light years. However, (b) is a conjecture based on (2). One of the important features of this cosmology is the requirement of spontaneous creation of matter. As the Universe expands, the mean density of a Universe will decrease, thus violating postulate (b). In order to maintain (b) it is necessary to postulate the spontaneous creation of matter. Thus, the steady state model requires the violation of the energy conservation principle.

Several revisions have been made to fit the theory to constantly accumulated observational facts. For some reasons, Hoyle finally gave up the theory.[24]

This theory requires a negative value of -1 for the deceleration parameter q_o. A negative value is allowed in this theory because the field equations are modified by nonconservation of energy.

The theory does not allow a uniform radiation background, thus contradicting the observed 3 ^oK cosmic background radiation.

(b) Dicke's scalar theory.[25]

In conjunction with Mach's principle, Dicke has postulated the existence of a scalar field whose source is the contracted stress energy tensor $T = \sum_i T^i_i$. According to this theory, the Universe can be made to close even for its present value of matter-energy density, due to galaxies alone. A consequence of the introduction of the scalar field is that the gravitational constant is a function of time, for or against which there is no definite observational data.

A gravitational theory with a scalar field will predict a different value for the advancement of the perihelion of planets and a different value for the bending of light. Unless it can be shown that there are other causes for the advancement of the perihelion of Mercury, the good agreement between theoretical prediction and observation must be taken as an evidence that the scalar field does not exist.[26]

The steady state theory and the scalar field theory will not be discussed further in this paper.

V. <u>Classical Tests of Cosmological Models</u>.[27]

As we have seen in the last section, the two important quantities which characterize a cosmological model are Hubble's constant H_o and the deceleration parameter q_o. H_o can be

$$m_{bol} = 5 \log \frac{1}{q_o^2} \left\{ q_o Z + (q_o - 1) \left[\sqrt{1+2 q_o Z} - 1 \right] \right\} + C, \quad q_o \neq 0 \qquad (26)$$

$$m_{bol} = 5 \log Z \left(1 + \frac{1}{2} Z\right) + C , \quad q_o = 0 \qquad (27)$$

This plot of the magnitude versus red-shift relation will therefore enable one to determine q_o . According to the most recent work, the magnitude - red-shift relation indicates a value of q_o very close to +0.5.[14] Figure 2 shows the most recent results of observations.

(ii) The Count-Magnitude Relation.

If galaxies are distributed uniformly in space, counts to successive limits of the parameter distance u , will yield numbers proportional to the volume inclosed within u . Because volumes in Riemannian geometry will either decrease slower or faster than u^3 , according to whether the curvature is positive or negative, a determination of the spatial curvature is possible in principle by counts. Letting N(m) be the number of galaxies brighter than the apparent magnitude m, Mattig obtained the following relations:[30]

$$N(m) = \frac{2\pi n}{Q H_o^3} \begin{cases} (1-2 q_o)^{-3/2} \left[P\sqrt{1+P^2} - \sinh^{-1} P \right] & k = -1 \\ (2 q_o - 1)^{-3/2} \left[\sin^{-1} P - P\sqrt{1-P^2} \right], & k = +1 , \end{cases} \qquad (28)$$

where n is the number-density of galaxies, and Q is the number determined directly from observation, but q_o is related to the matter-energy density of the Universe through the field equations. In order to compare theory with observation, it is necessary to obtain q_o from observations, either directly from the matter-energy density content of the Universe, or indirectly from the observation of distant galaxies. The classical test of cosmological theory involves obtaining q_o from the distribution, the size - red shift relation, and the magnitude - red shift relation of distant galaxies.

Information which can be extracted regarding the nature of the Universe is listed below.

(i) Magnitude versus Red-shift Relation.

As we have said before, the magnitude - red-shift relation can be used to check the validity of the inverse square law, and hence the geometric structure of the Universe. Without using the field equations, Heckman, Robertson and McVitte[28] obtained the following expansion:

$$m_{bol} = 5 \log Z + 1.086 \, (1 - q_o) \, Z + \dots \qquad (25)$$

which is correct to the first order in the red shift $Z = \frac{\Delta\lambda}{\lambda}$.
A method based on the assumption of field equations gives:[29]

of square degrees in the sky.

$$P = \frac{A\sqrt{k(2\,q_o - 1)}}{q_o(1 + A) - (q_o - 1)\sqrt{1+2\,A}} \;, \quad A = 10^{0.2(m_R - k_R - C')} \;, \tag{29}$$

where m_R is the magnitude in the red band, k_R is the k-correction term as explained in Section III, and C' is a constant 22.516.[27]

For the case $k = 0$, we find

$$N(m) = \frac{4\pi n A^3}{3Q\,H_o^3} \left\{ \frac{1}{2}\,(1 + A + \sqrt{1+2\,A}) \right\}^{-3} . \tag{30}$$

Figure 3 shows the difference in the count magnitude relation for various cosmological models. As seen, the count-magnitude relation can discriminate various cosmological models only for objects of magnitude of +23 or more. This is beyond the ability of the 200" telescope.

(iii) Angular diameters.

Because of curvature in space, the metric angular diameter of an object does not decrease linearly. For cases with $q_o \geq 0$, the relation is

$$\theta_o = \frac{C''(1 + Z)}{A} \;, \tag{31}$$

where A is given in Eq. (29).[31] Figure 4 shows the angular diameter red shift relation in different theories. It can be seen that for closed models the angular diameter will reach a minimum value, and then will increase. However, as Sandage pointed out, the isophotal radius will always decrease. The isophotal radius is the radius of contours of equal surface brightness of galaxies.* Because of red shifts, however, the surface brightness can change if the observation is made at a definite wave length. Hence, the isophotal radius is different from the metric angular diameter, which is the actual radius. The reason for this difference is the same as for the introduction of the k-term.

In conclusion, the usual tests of cosmological theory correlate the behavior of galaxies with large red shifts to the geometrical structure of space-time. These tests require the use of a large telescope, but at present the largest telescope, the 5-meter Hale telescope of Mt. Palomar, is somehow inadequate to discriminate between different cosmological models.[27]

VI. Proposed New Tests of Cosmological Models.

The most important property of a cosmological model is closure, which depends on the values of the Hubble constant and the matter-energy density of the Universe. The Hubble constant can be obtained directly from galactic research. In most cosmological work the closure property which is related to the

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*Because of Lionville's theorem, the surface brightness of an extended source does not change with distance. (Arking[31]) eration parameter q_o, is determined from observing distant galaxies. This approach has stimulated widespread research, including the present work. Here, however, we will use a different criterion for the acceptance of a cosmological model and the determination of the closure property of our Universe.

If we exclude certain cosmological models which contain a priori assumptions in violation with presently accepted physical laws, then it is unavoidable to admit that the present Universe originated from a singular state. A singularity of any kind is a clear violation of currently accepted concepts of physics, but the occurrence of this singularity may indicate the inadequacy of the classical theory of general relativity at too high densities.* Whatever the case may be, a theoretical singularity at the beginning of the Universe may indicate a very high density during and right after creation. If the present Universe evolved from a highly condensed state, then one can obtain information about the course of evolution of the Universe, based on the use of statistical physics and non-stationary processes.

Among all forms of matter-energy, the most likely ones which are of cosmological importance are the following:

(i) galaxies
(ii) radiation
(iii) intergalactic neutral atomic hydrogen

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*We do not mean that a quantized version of the General Relativity theory is the only answer, but we want to point out that classical general relativity is certainly inadequate at large densities.

(iv) intergalactic ionized hydrogen

(v) intergalactic hydrogen molecule

(vi) intergalactic neutral, ionized and second-ionized atomic helium

(vii) intergalactic atomic heavy elements (ionized or neutral)

(viii) neutrinos

(ix) gravitational radiation

(x) invisible forms of matter which include dust, rock-sized matter, black dwarfs (stars which are too small to be visible) dead galaxies, etc.

(xi) the existence of anti-matter in the Universe.

Observations on items (i), (ii) and (iii) are now available. It may be possible to obtain direct information on (iv), (v), (vi) and (vii) in the future. Observational information on (viii), (ix), and (x) are exceedingly difficult to obtain. However, if the universe were once in a highly condensed state and if a cosmological model is given, then, in principle, information on (iv) to (xi) can be obtained from statistical physics for non-equilibrium processes, in terms of information from (i), (ii) and (iii). These results can then be used to judge if the cosmological model in question is consistent with its basic assumption on any item from (iv) to (xi). Whenever additional information on any item from (iv) to (xi) is available, it can be used to further check the validity of the assumption of a singular origin of the Universe, by comparing the calculations with observations. In other words, we shall make definite predictions on (iv) to (xi). From these predictions we shall draw conclusions on the closure property of the Universe.

VII. Equilibrium Processes.[32]

In the following, we shall distinguish two cases, a weakly interacting gas and a non-interacting gas. In a weakly interacting gas it is assumed that the interaction is so strong that a thermodynamic equilibrium can be assumed and yet, at the mean time, so weak that the statistical properties are the same as an ideal gas. In a non-interacting case, the gas particles are assumed to be strictly non-interacting.

(i) Distribution functions for a gas in equilibrium.

In this case we are dealing with a weakly interacting gas. The distribution function $f(\epsilon)$ as a function of the gas particle kinetic energy ϵ [which $= (p^2 c^2 + m^2 c^4)^{\frac{1}{2}} - mc^2$, where p is the momentum] is

$$f(\epsilon) = \frac{1}{\exp \frac{\epsilon - \mu}{kT} \pm 1} \tag{32}$$

where the + sign in the denominator refers to fermions and the - sign refers to bosons, μ is the chemical potential and T is the temperature. The expressions of the pressure P, the internal energy U (not including the rest energy) and the particle number density n as functions of T and μ are as follows:

$$P = \frac{g}{6\pi^2 \hbar^3} \int_0^\infty f(\epsilon)\, p^3 \frac{\partial \epsilon}{\partial p}\, dp \tag{33}$$

$$n = \frac{g}{2\pi^2 \hbar^3} \int_0^\infty f(\epsilon)\, p^2\, dp \tag{34}$$

$$U = \frac{g}{2\pi^2 \hbar^3} \int_0^\infty f(\epsilon)\, \epsilon\, p^2\, dp \tag{35}$$

where g is the statistical weight.

For a dilute gas, the factor unity 1 in Eq. (32) can be neglected from the denominator, and the gas becomes a classical Maxwell gas.

Properties of P, n, U will be discussed in conjunction with specific problems involved.

(ii) Chemical equilibrium.

The condition for chemical equilibrium at constant temperature and pressure is given by

$$\sum_i \nu_i \mu_i - Q = 0 , \tag{36}$$

where Q is the energy released in the reaction, the subscripts i refer to the i-th component of the system. ν_i are integers with no common divisors, satisfying the chemical reaction

$$\sum_i \nu_i A_i = 0 , \tag{37}$$

where A_i is the notation for the i-th species.

(iii) Applications to a system of mixture of elementary particles.

(a) Pair Creation.

We now consider an equilibrium among protons, neutrons, electrons, photons and neutrinos. We shall neglect the nuclear reaction which leads to a build up of heavy elements. The reason for this omission is that the amount of heavy elements resulting from the initial step of nucleosynthesis is less than 30 percent.[33]

For a photon gas, the total number of photons is not fixed but is determined by the condition of equilibrium such that

$$\left(\frac{\partial s}{\partial n}\right)_{P,T} = 0 \tag{38}$$

where s is the entropy. Since $\left(\frac{\partial s}{\partial n}\right)_{P,T} = \frac{-\mu}{T}$, the chemical potential for a photon gas is zero. The chemical equilibrium condition for the pair creation reaction

$$\gamma - (A + \bar{A}) = 0 \tag{39}$$

is then

$$\mu_{\bar{A}} + \mu_A + 2mc^2 = 0 \quad \text{or} \quad \mu_A + mc^2 = -(\mu_{\bar{A}} + mc^2) \quad . \tag{40}$$

If the total number of particles (number of particles minus number of antiparticles) is conserved, then there is the subsidiary condition that

$$n_A - n_{\bar{A}} = n_o \tag{41}$$

where n_o is the number density of atomic electrons. Eqs. (40) and (41), together with Eq. (34), enable one to determine n_A and $n_{\bar{A}}$ in terms of the temperature T and ρ.

<u>Applications</u>: We now consider the case $n_o = 0$. We then find $\mu_A = \mu_{\bar{A}} = 0$, and

$$n_A = n_{\bar{A}} = \frac{g}{2\pi^2 \hbar^3} \int_o^\infty \frac{p^2\, dp}{\exp \frac{\varepsilon + mc^2}{kT} \pm 1} \tag{42}$$

$$U_A = U_{\bar{A}} = \frac{g}{2\pi \hbar^3} \int_o^\infty \frac{\varepsilon p^2\, dp}{\exp \frac{\varepsilon + mc^2}{kT} \pm 1} \ . \tag{43}$$

For the case of a photon gas, the particle and anti-particle are identical and $g = 2$. Therefore, we have

$$U_\gamma = aT^4 \tag{44}$$

$$n_\gamma = 2 \frac{k^3 T^3}{\pi^2 c^2 \hbar^3} \zeta(3) = 2.404 \frac{15}{\pi^4} \frac{a}{k} T^3 \tag{45}$$

$$P_\gamma = \frac{1}{3} U_\gamma = \frac{a}{3} T^4 \quad , \tag{46}$$

where a is the Steffan-Boltzmann constant and $\zeta(z)$ is the zeta function of Riemann.

For the case of a neutrino, since there is only one spin component for the observed neutrino and anti-neutrino respectively, g = 1. We find

$$U_\nu = U_{\bar{\nu}} = \frac{7}{16} a T^4 \tag{47}$$

$$P_\nu = \frac{1}{3} U_\nu \tag{48}$$

$$n_\nu = n_{\bar{\nu}} = 0.901 \frac{c^3 k^3}{\pi^2 \hbar^3} T^3 \quad . \tag{49}$$

Eqs. (47)-(49) are applicable to electron pairs and nucleon pairs at relativistic temperatures if every quantity is multiplied by a factor of 2.

For nonrelativistic temperatures such that $\frac{kT}{mc^2} << 1$, then we find

$$n_A = n_{\bar{A}} = \frac{m^3 c^3}{\pi^2 \hbar^3} \left(\frac{2}{\phi} \right)^{3/2} e^{-\phi} \sqrt{\frac{\pi}{4}} \tag{50}$$

$$P_A = P_{\bar{A}} = \frac{2}{3} (U_A) = \frac{2}{3} (U_{\bar{A}}) \tag{51}$$

$$(U_A + n_A mc^2) = (U_{\bar{A}} + n_{\bar{A}} mc^2)$$
$$= \left\{ \frac{15}{2^{3/2} \pi^{7/2}} \left(1 + \frac{3}{2\phi}\right) \phi^{5/2} e^{-\phi} \right\} a T^4 \tag{52}$$

where

$$\phi = \frac{mc^2}{kT} \quad . \tag{53}$$

(b) Ionization and molecular dissociation.

Consider the simple case of only one bound electron with no excited states. The reaction for ionization is:

$$A - (A^+ + e^- + \gamma) = 0 \quad . \tag{54}$$

Hence, the equation becomes

$$\mu_A - \mu_{A^+} - \mu_e - \mu_\gamma + E_i = 0 \tag{55}$$

where E_i is the ionization energy. Taking the nonrelativistic and the dilute gas limit in Eq. (34) we find,

$$\mu_A = kT \, \ln \left[\frac{n_A}{g} \frac{h^3}{(2\pi \, mkT)^{3/2}} \right] \quad . \tag{56}$$

Defining the degree of ionization α_i by

$$\alpha_i = \frac{n_+}{n_+ + n_A}$$

(α_i is the fraction of all atoms ionized), from Eq. (55) and (56) we obtain

$$\frac{\alpha_i^2}{1-\alpha_i} = \frac{g_{A^+}\ g_{e^-}}{g_A} \frac{1}{n_A + n_{A^+}} \left(\frac{2\pi\ m_e\ kT}{h^2}\right)^{3/2} e^{-E_i/kT} \ . \tag{57}$$

For molecules of type A_2 the degree of association α_d will be

$$\frac{\alpha_d^2}{1-\alpha_d} = \frac{g_A^2}{g_{A_2}} \frac{1}{n_{A_2} + \frac{1}{2}\ n_{A_2}} \left(\frac{\pi\ m_A\ kT}{h^2}\right)^{3/2} e^{-E_i/kT} \ . \tag{58}$$

If the excited states are taken into account, g_A is replaced by the partition function

$$Z_A = \sum_{\ell} g_{\ell} \exp - \frac{E_{\ell}}{kT} \ , \tag{59}$$

where the summation over ℓ is carried over all excited states, and E_{ℓ} is the energy of the excited state. Past experience shows that the inclusions of excited states do not change the simple results of Eq. (57) appreciably.

VIII. Non-Equilibrium Processes.[32]

The behavior of a system of particles not in thermodynamic equilibrium is described by the Boltzmann equation

$$\frac{\partial f_i(\underline{r}_i,\underline{p}_i,t)}{\partial t} + \underline{v}_i . \mathrm{grad}_{\underline{r}_i}\ f_i(r_i,\ p_i,t)$$
$$+ \dot{\underline{p}}_i . \mathrm{grad}_{\underline{p}_i}\ f_i(r_i,\ p_i,t) = \left[\frac{df_i}{dt}\right]_c \tag{60}$$

where r_i is the coordinate, $\underline{p}_i$ the momentum, $\underline{v}_i$ the velocity, and f_i the distribution function of the particle of the i-th species. In general f_i is time dependent, and is different from that for equilibrium process Eq. (32). $\left[\frac{df_i}{dt}\right]_c$ is the collision term which takes care of interactions. For a collision process, $\left[\frac{df_i}{dt}\right]_c$ is

$$\left[\frac{df_i}{dt}\right]_c = \sum_j \frac{g_j}{h^3} \int d^3\, p_i . \int d\Omega\ (\theta_i,\ \varphi_i')\ [f_i'\ f_i'\ (1 + \Theta_i\ f_i)(1 + \Theta_j\ f_j)$$
$$- f_i\ f_j\ (1 + \Theta_i\ f_i')(1 + \Theta_j\ f_j)]|\underline{v}_i , \underline{v}_j|\ \sigma(\theta_i',\varphi_i')\ , \tag{61}$$

where $\sigma(\theta_i',\ \varphi_i')$ is the scattering cross-section as a function of energy and the angle of scattering, $\theta_i',\ \varphi_i'$ of the i-th particle. $\Theta_i = +1$ and for fermions, $\Theta_i = -1$.

For cases where particle creation and destruction take place, we have

$$\left[\frac{df_i}{dt}\right]_c = \sum_j \left\{ \frac{g_i}{h^3} \int f_i' \, \xi^*_j \, (\underset{\sim}{p}_j', \underset{\sim}{p}_i) \, A_j \, (\underset{\sim}{p}_j', \underset{\sim}{p}_i)(1 + \Theta_j \, f_j) \right.$$

$$\left. \cdot \, (1+ \Theta_i \, f_i') \, d^3\underset{\sim}{p}_j' - \frac{g_i}{h^3} \int f_i f_j \, \sigma_a \, |\underset{\sim}{v}_i - \underset{\sim}{v}_j| \, (1 + \Theta_j f_j') \, d^3\underset{\sim}{p}_j \right\} \tag{62}$$

where $\xi^*_j \, (\underset{\sim}{p}_j', \underset{\sim}{p}_i')$ is the fractional number of particles of the j-th species and momentum p_j, which are capable of emitting a particle of the i-th species of momentum $\underset{\sim}{p}_i$, $A_j \, (\underset{\sim}{p}_j', \underset{\sim}{p}_i)$ is the rate of emission per emitter, and σ_a is the absorption cross-section. Eq. (62) is also applicable to annihilation processes, but in this case the factor $(1 + \Theta_i \, f_i')$ is absent in the second quantity on the right hand side of Eq. (62).

Eq. (60) is simplified when applied to cosmological models. If homogeneity and isotropy are assumed for the Universe, then $f_i(\underset{\sim}{r}_i, \underset{\sim}{p}_i, t)$ should not depend on $\underset{\sim}{r}_i$; the $\mathrm{grad}_{\underset{\sim}{r}_i}$ term therefore fanishes. The term $\underset{\sim}{p}_i$ is related to changes in the proper length ℓ, by the following relation

$$\frac{d \ln p}{dt} = - \frac{d \ln \ell}{dt} \tag{63}$$

This can be seen by the following argument, which is based on two assumptions: (a) locally, the space-time is flat and (b) the statistical properties of all similar systems are equivalent. The assumption (a) is a consequence of the geometrical character of space-time. It enables us to use the special relativistic theory of transport processes. The assumption (b) then enables us to consider an isolated system of particles in an expanding Universe.

Consider a cubic volume of sides of proper length R in an evolving Universe, where the sides are allowed to expand or contract with the Universe. Thus, as time passes, R will increase or decrease. If the box includes a large number of particles so that one can talk about the statistical properties of the system, then applying assumption (b), one can replace the boundaries of this box by perfectly reflecting walls, such that at the boundary of the volume, the momenta $p = (p_x, p_y, p_z)$ of all particles are reversed to $p' = (-p_x, p_y, p_z)$, $(p_x, -p_y, p_z)$, or $(p_x, p_y, -p_z)$. Because the sides of the cubic volume are allowed to expand with the Universe, the particle densities inside this box are the same as those outside. The size of R is so chosen that, $\frac{dR}{dt}$ is always small compared with the velocity of light.

Let the origin chosen be the center of the cube. Consider a particle whose momentum in the x-direction is p_x and whose velocity is v_x . In a time dt, the number of collisions of this particle with the wall is $\frac{R}{v_x}$. Because the walls are moving away with a velocity $\pm \frac{1}{2} \frac{dR}{dt}$ in the x and x-direction respectively, in each collision process this particle will lose momentum $\Delta p_x = m \frac{dR}{dt}$, where m is the relativistic mass. Therefore, in a time dt, the total amount of momentum lost is $- \frac{v_x}{R} m \frac{dR}{dt} dt = - p_x \frac{d \ln R}{dt} dt$. Thus, the rate at which momentum is lost is given by the equation

$$\frac{dp_x}{dt} = - p_x \frac{d \ln R}{dt} . \tag{64}$$

The same equation, of course, also applies to other components. The solution to Eq. (64) is

$$\frac{p_x}{p_x(o)} = \frac{R^{(o)}}{R} , \tag{65}$$

where the superscript (o) denotes initial values.

By denoting $\frac{d \ln R}{dt}$ by $H_o(t)$, (in cosmological models $H_o(t)$ is the Hubble's constant), because of Eq. (63), the transport equation then becomes

$$\frac{\partial f_i}{dt} - H_o(t) \underset{\sim}{p}_i \cdot \text{grad}_{\underset{\sim}{p}_i} f_i = \left[\frac{df_i}{dt} \right]_c . \tag{66}$$

Thus, cosmological effects enter into the transport equation only through the Hubble's constant $H_o(t)$.

IX. Non-interacting and Interacting Gases in Adiabatic Processes.

(i) Weakly interacting gas.

(a) Relativistic particles.

For a weakly interacting gas, thermodynamic equilibrium can be assumed, and for a relativistic gas $\epsilon = pc$. Eqs. (33), (34) and (35) become

$$P = \frac{g}{6}\left(\frac{kT}{\hbar c}\right)^3 kT \, f_2^{(\pm)}(\xi) \tag{67}$$

$$n = \frac{g}{2}\left(\frac{kT}{\hbar c}\right)^3 f_1^{(\pm)}(\xi) \tag{68}$$

$$U = \frac{g}{2}\left(\frac{kT}{\hbar c}\right)^3 kT \, f_2^{(\pm)}(\xi) \; ; \tag{69}$$

$$= 3\,P$$

where $\xi = \frac{\mu}{kT}$ and

$$f_n^{(\pm)}(\xi) = \int_0^\infty \frac{u^{n+1}\,du}{\exp(u-\xi)\pm 1} \; . \tag{70}$$

$f_n^{(+)}(\xi)$ is tabulated in Table 1 for several values of ξ and for cases $n = 1, 2$.

Theorem 1. For a relativistic gas, the parameter $\frac{\mu}{kT}$ remains constant in an adiabatic process, provided that the total number of particles remains constant.

Proof: If V is the volume of the system, then for adiabatic changes

$$PdV + d(UV) = 0 \quad . \tag{71}$$

Using the relation Eq. (69) to eliminate P, Eq. (71) gives

$$UV^{4/3} = \text{constant.} \tag{72}$$

If the total number of particles is constant, then we have

$$n\,V = \text{constant} \quad . \tag{73}$$

From Eqs. (71) and (72) we find that

$$\left[\frac{f_1^{(\pm)}\left(\frac{\mu}{kT}\right)}{f_2^{(\pm)}\left(\frac{\mu}{kT}\right)}\right]^{4/3} = \text{constant} \quad , \tag{74}$$

which is possible only if $\frac{\mu}{kT}$ is a constant.

<u>Corallary</u>. For radiation we have

$$P = \frac{a}{3}\,T^4 = \frac{1}{3}\,U \;. \tag{75}$$

Hence from the adiabatic condition Eq. (71) we find

$$TV^{1/3} = \text{constant} , \tag{76}$$

which is the same as Eq. (72) if we use the relation $U = aT^4$.

(b) Non-relativistic cases.

For non-relativistic cases $\epsilon = p^2/2m$. If the total number of particles is constant, then it is also true that

$$\frac{\mu}{kT} = \text{constant} , \tag{77}$$

$$UV^{5/3} = \text{constant} , \tag{78}$$

$$n\,V = \text{constant} , \tag{79}$$

and

$$TV^{2/3} = \text{constant} . \tag{80}$$

The proof is similar to that in Theorem 1.

(c) In general, in intermediate cases $\frac{\mu}{kT}$ is not a constant and the adiabatic laws become more complicated.

(d) If chemical reactions (pair creation, element equilibrium, and ionization) are taken into account, then in general, even for relativistic and nonrelativistic cases, the adiabatic conditions are more complicated.

One can define three adiabatic exponents Γ_1, Γ_2 and Γ_3 which describe the relation between P, V, and T. Let the two independent thermodynamic variables be denoted by x and y . We then have

$$\Gamma_1 = -\left(\frac{d\,\ell n\,P}{d\,\ell n\,V}\right)_{ad} = -\frac{V}{P}\,\frac{(P_x U_y - P_y U_x) + P(P_x V_y - P_y V_x)}{V_x U_y - V_y U_x}\;; \tag{81}$$

$$\frac{\Gamma_2}{\Gamma_2 - 1} = -\left(\frac{d\,\ell n\,T}{d\,\ell n\,V}\right)_{ad} = \frac{T}{P}\,\frac{(P_x U_y - P_y U_x) + P(P_x V_y - P_y V_x)}{(T_x U_y - T_y U_x) + P(T_x V_y - T_y V_x)} \tag{82}$$

$$\Gamma_3 - 1 = \left(\frac{d\,\ell n\,P}{d\,\ell n\,T}\right)_{ad} = -\frac{V}{T}\,\frac{(T_x U_y - T_y U_x) + (T_x V_y - T_y V_x)}{V_x U_y - U_x V_y} \tag{83}$$

For a non-interacting relativistic gas $\Gamma_1 = \Gamma_2 = \Gamma_3 = 4/3$ and for a non-interacting non-relativistic gas $\Gamma_1 = \Gamma_2 = \Gamma_3 = 5/3$. When phase transition takes place (e.g., electron pair creation, ionization, etc.), then Γ's will have lesser values, but the minimum value of Γ's is 1 for any reasonable gas.

(ii) <u>Strictly non-interacting gases</u>.

The case of a strictly non-interacting gas is of interest because when the temperature of the Universe is below say 10^9 ^oK, the relaxation time for establishing thermodynamic equilibrium for neutrinos is many times greater than the time scale of evolution of the Universe. Therefore a neutrino gas can and should be regarded as a strictly non-interacting gas. Further, at temperatures below 10^3 ^oK, radiation can also be regarded as a non-interacting gas in the further evolution of the Universe.

The transport equation for a non-interacting gas is

$$\frac{\partial f_i}{\partial t} - H_o(t)\ \underset{\sim}{p}_i \cdot \operatorname{grad}_{\underset{\sim}{p}_i} f_i = 0 \quad . \tag{84}$$

Eq. (84) will be used to demonstrate certain properties of a non-interacting gas.

<u>Theorem 2</u>. If initially the distribution function f_i is an equilibrium distribution function, then it will remain so in the relativistic and non-relativistic cases.

Proof: First consider the relativistic case. The equilibrium distribution function f is

$$f = \frac{1}{\exp \frac{\varepsilon-\mu}{kT} \pm 1} \tag{85}$$

for fermion cases (+) and boson cases (-), respectively. Letting $\Phi = \frac{\varepsilon-\mu}{kT}$, the transport equation then becomes

$$\frac{\partial f}{\partial \Phi}\frac{\partial \Phi}{\partial t} - H_o(t)\ \frac{\partial f}{\partial \Phi}\ p \cdot \frac{\partial \varepsilon}{\partial \underset{\sim}{p}}\frac{1}{kT} = 0 \ , \tag{86}$$

or

$$\frac{\partial \Phi}{\partial t} - H_o(t)\, \underset{\sim}{p} \cdot \frac{\partial \epsilon}{\partial \underset{\sim}{p}} \frac{1}{kT} = 0 . \tag{87}$$

Since $\underset{\sim}{p} \cdot \frac{\partial \epsilon}{\partial \underset{\sim}{p}} = pc = \epsilon$, we find

$$\epsilon \frac{\partial}{\partial t} \frac{1}{kT} - \frac{\partial}{\partial t}\left(\frac{\mu}{kT}\right) - \epsilon\, H_o(t) \frac{1}{kT} = 0 . \tag{88}$$

μ is determined from the number density of particles. Let us <u>assume</u> that $\frac{\mu}{kT}$ is constant. Then, Eq. (88) can be solved to give

$$T = T^{(o)} \frac{R^{(o)}}{R} . \tag{89}$$

This means that the concept of temperature is valid. We have previously shown that $p \,\alpha R^{-1}$ (Eq. 65)). Hence, f is invariant. We therefore find

$$TV^{1/3} = \text{constant} \tag{90}$$

$$PV^{4/3} = \frac{1}{3} UV^{4/3} = \text{constant} \tag{91}$$

and

$$nV = \text{constant} , \tag{92}$$

which satisfies the condition that $\frac{\mu}{kT}$ = constant. (Theorem 1). Thus, in an adiabatic process for a relativistic non-interacting gas, the spectrum will remain unchanged and the usual adiabatic laws will apply.

The proof for the non-relativistic case is similar. The relation between T and R is

$$T = T^{(o)} \left(\frac{R^{(o)}}{R}\right)^2 , \quad \frac{\mu}{kT} = \text{constant} . \tag{93}$$

Hence,

$$nV = \text{constant} \tag{94}$$

$$PV^{5/3} = \frac{2}{3}\, UV^{5/3} = \text{constant} \tag{95}$$

$$TV^{2/3} = \text{constant} \tag{96}$$

For semi-relativistic cases, the temperature cannot be defined and the spectrum is no longer invariant. This stems from the absence of a simple power law dependence of ϵ on p in semi-relativistic cases.

In conclusion, in an expansion or contraction process, for the case of a strictly non-interacting gas in the relativistic and non-relativistic limit, if the distribution function is initially in equilibrium, it will remain so in the course of time. Further, if the number of particles is conserved, then the ratio $\frac{\mu}{kT}$ remains constant. The spectrum also remains invariant.

X. Annihilation of Particle Pairs in an Evolving Universe.

We now consider the problem of annihilation of particle pairs in an evolving Universe. This problem arises from the belief that, since particles and antiparticles show a symmetry in their properties, it is possible that this symmetry property is also reflected in the over-all particle-antiparticle population of the Universe.[34]

If the Universe has a singular origin, then at some time in the past the temperature and density of the Universe must be so high that particle pairs (including nucleon pairs) are in equilibrium with radiation. If equilibrium holds to very low temperatures, then all nucleon pairs will annihilate. However, as the Universe expands, conditions for achieving statistical equilibrium may not be satisfied, and nucleon pairs may survive annihilation during the expansion.

The collision term $\left[\frac{df}{dt}\right]_c$ for the annihilation process is

$$\left[\frac{df}{dt}\right]_c = - \frac{\bar{g}}{h^3} \int d^3 \bar{\underset{\sim}{p}} \int \sigma \, v \, \bar{f} \, (\bar{\underset{\sim}{p}}) \, f \, (\underset{\sim}{p}) + \text{Production rate} \qquad (97)$$

where σ is the annihilation cross section, v is the relative velocity, and an overbar denotes quantities associated with anti-particles. A similar expression can be obtained for $\left[\frac{d\bar{f}}{d\epsilon}\right]_c$.

The production rate is usually a complicated function of temperature and density. Therefore, instead of considering the full problem, we can consider the case where the initial particle density is many times greater than the equilibrium pair density. This will mean that the production rate is small compared with the annihilation rate. The transport equation then becomes:

$$\frac{\partial f}{\partial t} - H_o(t)\ p\ .\ \frac{\partial f}{\partial p} = -\ f\bar{N}(\sigma v) \tag{98}$$

$$\frac{\partial \bar{f}}{\partial t} - H_o(t)\ p\ \frac{\partial f}{\partial p} = -\ \bar{f}N(\sigma v) \tag{99}$$

At low energy, σv is a constant. After multiplying Eqs. (98) and (99) by $\frac{2}{h^3} d^3 \underset{\sim}{p}$ and $\frac{2}{h^3} d^3 \underset{\sim}{\bar{p}}$, and integrating over $d^3 \underset{\sim}{p}$ and $d^3 \underset{\sim}{\bar{p}}$ respectively, we obtain:

$$\frac{\partial N}{\partial t} + 3\ H_o(t)N = -\ N\bar{N}(\sigma v) \tag{100}$$

$$\frac{\partial \bar{N}}{\partial t} + 3\ H_o(t)\bar{N} = -\ N\bar{N}(\sigma v) \tag{101}$$

where N and $\bar{N}$ are the number densities of particles and anti-particles. Equating Eqs. (100) and (101), we find

$$\frac{\partial(N-\bar{N})}{\partial t} + 3\, H_o(t)\,(N-\bar{N}) = 0\ . \tag{102}$$

The solution is

$$\ell n(N-\bar{N}) = -\int 3\, H_o(t)\; dt + \text{constant}\ , \tag{103}$$

or, using the definition $H_o(t)$ (Eq. (19)), we find

$$(N-\bar{N})\; R^3 = \text{constant} \tag{104}$$

which states that the total number of particles is conserved (cf. conservation of baryons). Rewriting Eq. (103):

$$N-\bar{N} = \Delta N_o\; \exp - \int 3\, H_o(t)\; dt \tag{105}$$

where ΔN_o is a constant. By substituting Eq. (105) into Eq. (100) we now obtain

$$\frac{\partial\bar{N}}{\partial t} + \left[3\, H_o(t) + \Delta N_o\; \sigma v\; \exp \int 3\, H_o(t)\; dt\right] \bar{N} = -\,\sigma v\; \bar{N}^2 \tag{106}$$

Eq. (106) can be linearized by the substitution $\bar{N} = Z^{-1}$:

$$\frac{\partial Z}{\partial t} - \left[3\ H_o(t) + \Delta N_o\ \sigma v\ \exp - \int 3\ H_o(t)\ dt\right] Z = \sigma v \tag{107}$$

The solution is

$$Z = C_1\ e^{\int h(t)dt} + e^{\int h(t)dt} \int g(t) e^{\int -h(t)dt}\ dt \tag{108}$$

where $g(t) = \sigma v$

$$h(t) = 3\ H_o(t) + \Delta N_o\ \sigma v\ \exp - \int 3\ H_o(t)\ dt$$

and the arbitrary constant C_1 is to be determined from the actual member density N at a certain time t. Eq. (108) is a general solution, valid for any cosmological model.

XI. Co-existence of Particles and Anti-particles in an Evolving Cosmological Model.[35]

We now apply Eq. (108) to an evolutionary cosmological model. For a Universe in which radiation energy is dominant, then,

$$H_o(t) = \frac{1}{2t} \tag{109}$$

From the thermodynamic relation that $TV^{4/3}$ = constant, we find that

$$T(t) = T'\ t^{-\frac{1}{2}} \tag{110}$$

and, if the total number of particles is conserved,

$$n(t) = n' \, t^{-3/2} \; . \tag{111}$$

Different cosmological theories will give somewhat different values of T' and n'. Alpher et al[36] gave $T' = 1.5 \times 10^{10}\ {}^{o}K$. The evaluation of n' is different from the evaluation of T'. The present universe is a matter universe for which

$$H_o(t) = \frac{2}{3}\frac{1}{t} \tag{112}$$

$$T\,(t) \propto t^{-2/3} \tag{113}$$

$$n\,(t) \propto T^3 \propto t^{-2} \tag{114}$$

Hence n' must be evaluated from the equation

$$n\,(t) = n' \left(\frac{T}{T'}\right)^3 \; . \tag{115}$$

Using the present value of $n = 10^{-7}$ and the constant T', one finds that $n' = 10^{22}$, if the age of the Universe is taken to be $\sim 10^{10}$ years. The proper inclusion of interactions between particles and anti-particles will only slightly alter the numerical character of the result. Substituting Eq. (109) into Eqs. (105) and (108), we then find

$$N - \bar{N} = \Delta N_o \, t^{-3/2} \tag{116}$$

$$\bar{N} = \frac{\bar{N}^{(o)} \left(\frac{t}{t_o}\right)^{-3/2}}{(1 + \bar{N}^{(o)} \, t_o^{3/2}/\Delta N_o) \exp\left\{ \frac{2\Delta N_o \, \sigma v}{t_o^{\frac{1}{2}}} \left[1-\left(\frac{t_o}{t}\right)^{\frac{1}{2}}\right] \right\} - \frac{\bar{N}^{(o)} t_o^{3/2}}{\Delta N_o}} \tag{117}$$

where $\bar{N}^{(o)}$ is the value of $\bar{N}$ at $t = t_o$. In the case $N = \bar{N}$, it is easily shown that the solution is

$$N = \bar{N} = \frac{N^{(o)}}{(1 + 2 \, \sigma v \, \bar{N}^{(o)} t_o)\left(\frac{t}{t_o}\right)^{3/2} - 2 \, \sigma v \, t_o \, \bar{N}^{(o)}\left(\frac{t}{t_o}\right)} \tag{118}$$

Comparing Eq. (118) with Eq. (111), we see that in the case $\bar{N} = N$, the pair density is quickly reduced by a depletion factor $1 + 2 \, t_o/\tau_a$ after a time of the order of t_o has elapsed. $\tau_a = [\sigma v \, \bar{N}^{(o)}]^{-1}$ is the mean life time of a particle (or an anti-particle) against annihilation at $t = t_o$. In the case of Eq. (117), however, the depletion factor is

$$\left[1 + \frac{\bar{N}^{(o)} \, t_o^{3/2}}{\Delta N_o}\right] \exp\left(\frac{2\Delta \, N_o \, \sigma v}{t_o^{\frac{1}{2}}}\right) - \frac{\bar{N}^{(o)} \, t_o^{3/2}}{\Delta \, N_o}$$

which can be very large if $\frac{\Delta n' \sigma v}{t_o^{1/2}} >> 1$.

Now we apply Eqs. (117) and (118) to our Universe. First we must find a set of the initial density and temperature of the Universe at the earliest possible epoch for which the assumptions underlying Eqs.(117) and (118) are still valid. We choose $t_o = 0.01$ sec. The initial temperature and density computed from Eqs. (110) and (115) are 1.5×10^{11} °K and $10^{25}/\text{cm}^3$ respectively. In the nonrelativistic limit, the equilibrium proton pair density $n^{(e)}$ is given by (see Eq. 34))

$$n^{(e)} = \frac{\pi}{4} \frac{m_p^{\ 3} c^3}{\pi^2 \hbar^3} \left(\frac{2}{\Phi}\right)^{3/2} e^{-\Phi} = 1.2 \times 10^{38}\ T_{11}^{\ 3/2}\ e^{-\frac{109}{T_{11}}},\quad T_{11} = T/10^{11}\ {}^{\circ}\text{K} \tag{119}$$

where $\Phi = \frac{m_p c^2}{kT} = \frac{1.09 \times 10^{13}}{T}$, and m_p is the proton mass. From Eq. (119) we find that at $T = 1.5 \times 10^{11}$ °K, the equilibrium density $n^{(e)}$ is much less than the initial density $10^{25}/\text{cm}^3$ so that the use of our theory is justified. But this temperature is not very different from the temperature of 2×10^{11} °K at which the equilibrium density is also $10^{25}/\text{cm}^3$. This is due to the steep temperature dependence of the pair density. The value of σv for p-$\bar{p}$ annihilation at zero energy is a constant with $\sigma v = 2 \times 10^{-15}$ cm^3/sec.[37] We therefore find that, in the case of Eq. (118), the depletion factor $1 + 2\ t_o/\tau_a$ is of the order of 10^8. Thus, had our Universe been created with an equal number of particles and anti-particles, most pairs would have annihilated in the first few hundredths of a second after creation, and the present Universe would have been richer in radiation energy than what is observed. Further, unless there is a mechanism not yet known to us which can cause a complete separation between particles and anti-particles to take place before the formation of galaxies, an even larger fraction of nucleon pairs will annihilate during galactic and star formation, resulting in a number of 75 MeV photons from the decay of π^o -mesons produced in p-$\bar{p}$ annihilations. Since the present observations exclude such a high energy flux, we conclude that the assumption of an equal number of particles and anti-particles at the time of creation of the Universe is incompatible with both observations and presently accepted physical laws and world models.

If the Universe is initially richer in particle population, then Eq. (117) tells us that as long as $\Delta n' \sigma v/t_o^{\frac{1}{2}} >> 1$, virtually all anti-particles created at the early epoch will have annihilated in the first few hundredths of a second after creation. The present Universe is therefore free of any residual anti-particle associated with the creation of the Universe. This is consistent with the available data on cosmic ray anti-proton fluxes. The present calculation thus precludes the presence of anti-galaxies.

XI. Estimates of the Upper Limit of Matter-Energy Density of the Universe.

Eq. (118) tells us that the depletion factor is of the order of t_o/τ, where τ is the mean interaction time $(\sigma VN)^{-1}$. We now apply this criteria to obtain rough estimates of all forms of matter energy in terms of observed radiation density and matter density (galaxies and intergalactic hydrogen). An exact treatment of this problem with a better cosmological model will be discussed in a forthcoming paper.

(1) Intergalactic ionized hydrogen.

According to Alpher's model Universe,[36] down to a temperature of 300 ^oK radiation, energy is dominant. We can use Eq. (115) for the temperature-density relation:

$$n = 10^{22} \left(\frac{T}{1.5 \times 10^{10}}\right)^3 \quad . \tag{120}$$

Substituting Eq. (120) into Eq. (57), we obtain the degree of ionization of hydrogen α_i in terms of the temperature of the Universe:

$$\frac{\alpha_i^2}{1-\alpha_i} = 1.4 \times 10^{16} \left(\frac{1.57 \times 10^5}{T}\right)^{3/2} \exp - \frac{1.57 \times 10^5}{T} \quad . \tag{121}$$

When the value of the quantity on the left hand side is of the order of unity, ionization equilibrium will begin to shift towards neutral hydrogen. This takes place at $T = 3400\ ^{o}K$, where the corresponding density is $n \sim 100$ and $t_o = 10^{13}$ seconds. The recombination cross-section is $10^{-20}\ cm^2$ at this temperature, and hence, $\sigma V = 4 \times 10^{-13}$ and $\tau = 7 \times 10^{10}$ seconds, which is much smaller than t_o. <u>We may therefore conclude that most ionized hydrogen will recombine at approximately 10^{13} seconds after creation</u>. The question is then whether intergalactic neutral hydrogen can become ionized after the formation of galaxies. Gould and Ramsey[38] have shown that if the intergalactic hydrogen is not ionized initially before galactic formation, then it is unlikely that an ionized state can come into being afterwards and be maintained at present. Further, down to a temperature of $300\ ^{o}K$, radiation is dominant. This means that the total energy (self gravitational energy plus the thermal energy) of a region in space is always close to zero. The necessary condition for gravitational condensation is that there exist regions in space for which the total energy is negative. Therefore galaxies cannot form until radiation energy is no longer dominant. This means that the recombination of hydrogen will take place long before galactic formation. Hence, based on our result and Gould and Ramsey's estimate we can conclude tentatively that the presence of a large amount of intergalactic ionized hydrogen is basically inconsistent with the accepted principle of physics and the singular origin of the Universe. A better calculation will tell us the upper limit of the amount of ionized hydrogen in terms of the total intergalactic neutral hydrogen and the total mass in galaxies.

(ii) Intergalactic hydrogen molecule.

Using Eq. (58) and Eq. (120) we find that

$$\frac{\alpha_d^2}{1-\alpha_d} = 1.9 \times 10^{23} \left(\frac{5.2 \times 10^4}{T}\right)^{3/2} e^{-5.2 \times 10^4/T} , \tag{122}$$

At $\alpha_d \sim \frac{1}{2}$, we find $T = 870\ ^\circ K$ with the corresponding value of $n = 2$ and $t_o = 3 \times 10^{14}$ sec. The direct recombination of two hydrogen atoms in their ground state into a hydrogen molecule ($2H \rightarrow H_2$) is a very improbable process because the transition is a forbidden one.[39] The recombination can take place through one of the excited states (e.g., one atom in the ground state and the other 2) state) or via a three body process $3H \rightarrow H_2 + H$. The fractional number of atoms in the first excited state at $T = 847\ ^\circ K$ is roughly $e^{-100} \approx 10^{-43}$, and hence can be entirely neglected. The transition probability for the three body process $3H \rightarrow H_2 + H$ is roughly 10^{-32} cm^6 sec^{-1},[40] which is too small to yield any significant recombination reactions because of the low density.

Gould and Salpeter[39] suggested that in interstellar space, recombination may take place on the surface of dust grains. However, as we shall see below, the dust grains are not likely to exist before galactic formation and before the temperature of the Universe has dropped substantially. Hence we conclude that very little hydrogen will become molecules in the early evolution of the Universe. Because the density of hydrogen in intergalactic space will always decrease, the formation of hydrogen molecules in intergalactic space will become even more unlikely after galactic formation. Thus it appears that there are very few intergalactic hydrogen molecules. We may remark that the presence of a large amount of intergalactic molecular hydrogen should yield some absorption lines at the first ionization potential of 15.4 eV in distant quasars and should be detectable.

Based on (1) and (2) and the observed upper limit of intergalactic neutral hydrogen density, we conclude that almost all hydrogen appears to be located in galaxies.

(iii) Intergalactic heavy elements.

The abundances of heavy elements (He and others) have been estimated by various groups, with the most optimistic result ascribing no more than 20 per cent of all masses to the heavy elements.[33] Therefore, at best, 20 per cent of matter will be in the form of elements other than hydrogen.

(iv) Neutrinos

It has been conjectured from time to time that the neutrino energy density of the Universe can be very large.[41] As we have seen, if the total number of neutrinos is equal to that of anti-neutrinos and if a state of thermodynamic equilibrium is maintained at the early epoch, then the neutrino energy density is of the same order of magnitude as the photon energy density, and the spectrum of the neutrino gas cannot be very different from the black body neutrino spectrum. At $t_o = 10^{-6}$ sec, when $T \cong 10^{13}$ °K, the matter density is around 10^{40} particles/cm^3. The interaction cross section of neutrinos is around 10^{-38} cm^2. Hence, the interaction time $\tau \sim (\sigma VN)^{-1}$ is 10^{-12} sec. A state of thermodynamic equilibrium must therefore exist for neutrinos at the initial evolutionary phase of the Universe.

In order that neutrino density may be much larger than that of photon density, it is necessary that a degenerate state exist. Being a relativistic gas, according to Theorem 2, $\frac{\mu}{kT} = \frac{E_F}{kT}$ must be a constant, where E_F is the fermi energy of the neutrinos. If E_F is different from zero, the population of neutrinos must greatly exceed that of anti-neutrinos or vice versa. Let us assume that the anti-neutrinos are more abundant. (The calculation with a more abundant neutrino population is the same.) Then, from Eq. (69), we find that the neutrino energy density U_ν is

$$U_\nu = 4\pi c \left(\frac{k}{hc}\right)^4 T^4 \, f_2^{(+)} \left(\frac{E_F}{kT}\right) = 5.824 \times 10^{-16} \, T^4 \, f_2^{(+)} \left(\frac{E_F}{kT}\right) \; .$$

Using the present value of $T = 3\ {}^{\circ}K$, we find that if U_ν is to be equated to 10^{-29} g/cm, it is necessary that

$$f_2^{(+)} \left(\frac{E_F}{kT}\right) = 5.74 \times 10^5$$

10^{-29}g/cm is the energy density necessary for closure. For comparison, the present matter density due to galactic masses = 7×10^{-31} ergs/cm^3. From Table 1 we find that

$$\frac{E_F}{kT} = 40 \; ,$$

so that the value of the present fermi energy will be of the order of 1.6×10^{-2} eV, which is too small to be detected. However, the excessive number density of anti-neutrinos over neutrinos at present is of the order of $n_{\bar{\nu}} = 1.406 \left(\frac{E_F}{kT}\right)^3 T^4 = 7 \times 10^5$. Comparing this with the present density of protons, we see that the total number of anti-neutrinos is around 10^{13} times that of electrons or protons. We must ask: Is there any particular reason that there should be such an excessive number of neutrinos (or anti-neutrinos) in the Universe?

If the number of excessive anti-neutrinos is equated to the number of electrons (also equated to protons) then $\frac{E_F}{kT} \approx 0$ and the neutrino energy density is close to the black body radiation energy density.

We therefore conclude that neutrinos are not likely to contribute much to the overall matter energy density of the Universe.

(v) Gravitational Radiation.

Because the gravitational field is a tensor field, (gravitational quantum) a graviton has a spin two; and because the static gravitational force is an inverse square force, the rest mass of a graviton is zero. Therefore, gravitons in thermal equilibrium with their surroundings must have the same spectrum as radiation, and subsequently, the same energy density-temperature relation. The question then is whether thermal equilibrium can exist. According to Eq. (111), the number density is

$$n(t) = n' \ t^{-3/2} \quad .$$

Hence,

$$\tau = (\sigma v n)^{-1} = \frac{t^{3/2}}{\sigma v n'}$$

The average energy of the graviton is proportional to T. In order that τ should decrease as t decreases, (σV) as should not depend on the energy faster than (energy of graviton) where $k > -\frac{1}{2}$. (Note, for neutrinos, k is positive). When this condition is fulfilled, then there is always a value of t for which $\tau < t$, so that thermal dynamic equilibrium can be achieved. However, according to Weber,[43] the scattering cross-section of a graviton by a material particle which is damped only by re-radiation is proportional to λ^2 or E^{-2}, and one which is damped by other irreversible processes (e.g., electromagnetic radiation) is proportional to E^{-1}. Hence, there may be some question as to whether gravitational radiation may come into equilibrium with matter at an early epoch. However, gravitational radiation comes into equilibrium with macroscopic matter (galaxies, etc.) at a later stage, since the cross-section increases with wave length.

(vi) Invisible Form of Matter.

Other forms of invisible matter can be classified under two categories: (i) those bound by chemical forces (e.g., rocks, dust, etc.); and (ii) those formed by their own gravitational field (small planets, black dwarfs, etc.).

(1) Intergalactic dusts from primordial time.

No matter can exist in the form of a solid at temperatures above a few thousand degrees Kelvin. The formation of dust and rocks can take place only when the temperature is low so that the vapor pressure is comparable to the pressure in the Universe. The composition of dust grains has been speculated to be icy crystals and other impurities. At $T \sim 300\ ^{o}K$ the vapor pressure of metals (e.g., copper) is around 10^{-14} mm of mercury. This will correspond to a particle density of 10^{6} particles/cm^{3}, which is several orders of magnitude greater than the actual density of the Universe at this temperature. Hence even heavy metals cannot condense to become dust particles. For ice, the vapor pressure will be even greater, and hence dust can be formed only at very low temperatures. We therefore conclude that only a negligible fraction of primordial matter can condense into dust.

(2) Black dwarfs.

The next question is whether there are invisible objects bound by their own gravitational force in the intergalactic space. The condition for gravitational binding for a gas cloud of mass m is that its total energy must be less than zero. Assuming a uniform density ρ and a uniform temperature T, the dimension of the gas, R, must be such that

$$\frac{4\pi}{3} G \rho^2 R^5 \left[1 - \left(\frac{R_g T}{G_\rho}\right)^2 \right] > 0$$

$$R > R_o = \left(\frac{9}{8\pi}\frac{R_g T}{G_\rho}\right)^{\frac{1}{2}}$$

where ρ is the density, R the dimension, and T the temperature of the gas. Thus conditions for the formation of a large object (such as a galaxy) are more favorable than the conditions for the formation of a small object, for a given temperature and density. Therefore, it is unlikely that there are large masses condensing to become invisible astronomical objects before galactic formation in the intergalactic space. We negject from our discussion the possibility of the existence of a large amount of matter in collapsed state

(vii) Shape of the spectrum of the cosmic radiation background.

During hydrogen recombination it is possible that the spectrum of photons may be altered by free-free, free-bound processes. However, since the number of photons is many orders of magnitude greater than the number of hydrogen atoms, when recombination takes, it is also unlikely that the photon-spectrum will be seriously altered. Hence we believe that the spectrum of the background to radiation must be that of a black body to a high degree of approximation. This will be discussed in a future paper.

Weymann(43) has considered the shape of the cosmic radiation background. However, he assumed a very high density of matter ($\sim 10^{-29}$ g/cm^3) which is incompatible with observations, as we have shown in this paper. His result is still interesting because under some circumstances the black body spectrum is substantially altered.

XIII. Discussion.

The development of general relativity has been regarded as one important step towards understanding the structure and evolution of the Universe. Its success in predicting the second order corrections to particle trajectories and light rays in a gravitational field, marks a triumph in understanding the small (but not microscopic) scale of behavior of the gravitational field. Its success is further glorified by its prophecying the expansion of the Universe, which is observed later. Yet in the meantime, a theory as complicated as general relativity, predicted a remarkably limited number of cosmological models. In its unadulterated version, the theory allows only two types of cosmological models. One contains an infinite amount of matter, originated from a singular origin, and destined to expand until oblivion (open Universe); and one contains a finite amount of matter, also originated from a singular origin, which will expand to a maximum dimension, and then contract back to a singular state (closed Universe). The behavior of a particular model depends on two parameters: the overall matter-energy density of the Universe and the Hubble's constant.

For our Universe, the Hubble's constant as determined from galactic research is in the neighborhood of $(3.3 \times 10^{17}\ \text{sec})^{-1}$.

The matter-energy density attributed to galaxies is in the neighborhood of 7×10^{-31} g/cm^3. This is too small for closure.

For the reason that Mach's principle is not applicable to an open model, most cosmologists prefer to have a closed model. In order to do so, cosmologists generally choose one of the two alternatives:

(a) that there is a large amount of matter-energy in invisible form

(b) that some physical laws are violated. Invariably, the laws to be violated and the degree of violation, are so chosen that results are many orders of magnitude beyond the present technological limit of verification or disreputation.

In this paper we have shown that if we apply to our Universe cosmological models which have a singular origin, then the bulk of matter-energy density is in the form of galaxies. Hence we have shown that (a) cannot be true.

We regard the alternative (b) as totally unsatisfactory. To postulate and to accept a law because it cannot be shown to be wrong, without, in the meantime, suggesting some method of experimental confirmation, is the same as to accept the book of Genesis as the scientific theory of creation, on the basis that it cannot be refuted. We believe that a physical law has no right to exist unless it can be shown otherwise.

The necessity for postulating (a) and (b) stems from the belief that Mach's principle has the supreme command in cosmological theories. However, the fact that Mach's principle cannot be formulated unambiguously is a reason for not adopting it at present. Even as a principle, as Zeldovich pointed out,[44] it lacks a unity. Once we free ourselves from Mach's principle, we can study the Universe as it is. Unless it can be shown that Mach's principle is necessary, we should not still worry about it.

There are still some unanswered questions; among them are:

(a) Does the existence of a singularity in all cosmological models mean that a new theory of gravitation is required at high density?

(b) Why in our Universe does only one type of particle population dominate? Does this mean the high symmetry between particles and anti-particles breaks down at very high density?

We have no idea what the answers are, nor in which direction we must search for an answer.

XIV. A Cosmological Model for our Universe.

Based on information available to us, and assuming the validity of all physics laws, we conclude that the bulk of matter-energy in our Universe is in the form of galaxies. Our Universe can be described by an open model with $q_o = +0.02$.

Sandage[14] has assigned a value of +1.65 to q_o. This value is obtained using brightest members of clusters as samples. This value of q_o must be corrected for galactic evolution, since the galaxies which are responsible for the large value of q_o are younger than our galaxies by a few billion years. After these corrections are taken into account, the value of q_o is brought down to +0.5. No uncertainty has been assigned to this value of q_o .

The discrepancy between Sandage's value and ours is not serious, in view of difficulties of determination of q_o. Our value of q_o essentially is proportional to the density of matter due to galaxies. The value quoted here, 7×10^{-31} g/cm^3, was given by Oort in 1958. In view of the importance of the density of matter in galaxies, a new determination will be of vital interest and great importance.

<u>In conclusion, the structure of our Universe is consistent with an open model with a value of q_o between 0.02 and 0.5</u>.

XV. Acknowledgements.

I would like to thank Drs. R. Stothers and K. Thorne for discussions.

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TABLE I. $f_s^{(+)}(\xi) = \int_0^\infty \frac{x^{s+1}\,dx}{\exp(x-\xi)+1}$

ξ	s = 1	s = 2
0	1.80306	5.68213
0.2	2.16113	6.86839
0.4	2.58313	8.28826
0.6	3.07823	9.98279
0.8	3.65637	11.9988
1.0	4.32827	14.3891
1.4	5.99942	20.5390
1.8	8.18972	28.9952
2.2	11.0056	40.4445
2.6	14.5593	55.7036
3.0	18.9683	75.7289
3.4	24.3531	101.618
3.8	30.8363	134.616
4.2	38.5430	176.115
4.6	47.5984	227.658
5.0	58.1286	290.941
6.0	91.7432	512.999
7.0	137.363	853.407
8.0	198.984	1351.18
9.0	272.607	2051.32
10.0	366.228	3004.82
12.0	615.475	5905.92
14.0	960.718	10582.5
16.0	1417.96	17658.6
18.0	2003.21	27854.1
20.0	2732.45	41985.0
>20	$\cong \frac{1}{3}\xi^3$	$\cong \frac{1}{4}\xi^4$

FIGURE CAPTIONS

Fig. 1. Measurement of cosmic radiation in the microwave region (after Thaddeus).

Fig. 2. The theoretical red-shift versus magnitude relation. Magnitudes are the V-magnitude (average intensity around a central wave length of 5400 Å). Data for 18 clusters of galaxies are plotted as given by Humason, Mayall, and Sandage. Arrows are placed at the observed red-shift values for three distant clusters whose magnitudes are not quite available.[27]

Fig. 3. Comparison of the count-magnitude relation for the steady state model (s.s.) and the two exploding models $q_o = \frac{1}{2}$ and $q_o = 2\frac{1}{2}$. Note the extreme small difference in the predicted log N(m) values between the three models for magnitudes lighter than $m_R - K_R = 23$. Values of the red shift z are shown on each curve.[27]

Fig. 4. Metric diameters (in arbitrary units) of either individual galaxies or of clusters of galaxies computed on the magnitude system of the brightest cluster galaxy. Line of constant red shifts are shown. The straight line gives the <u>isophotal</u> diameters for all models.[27]

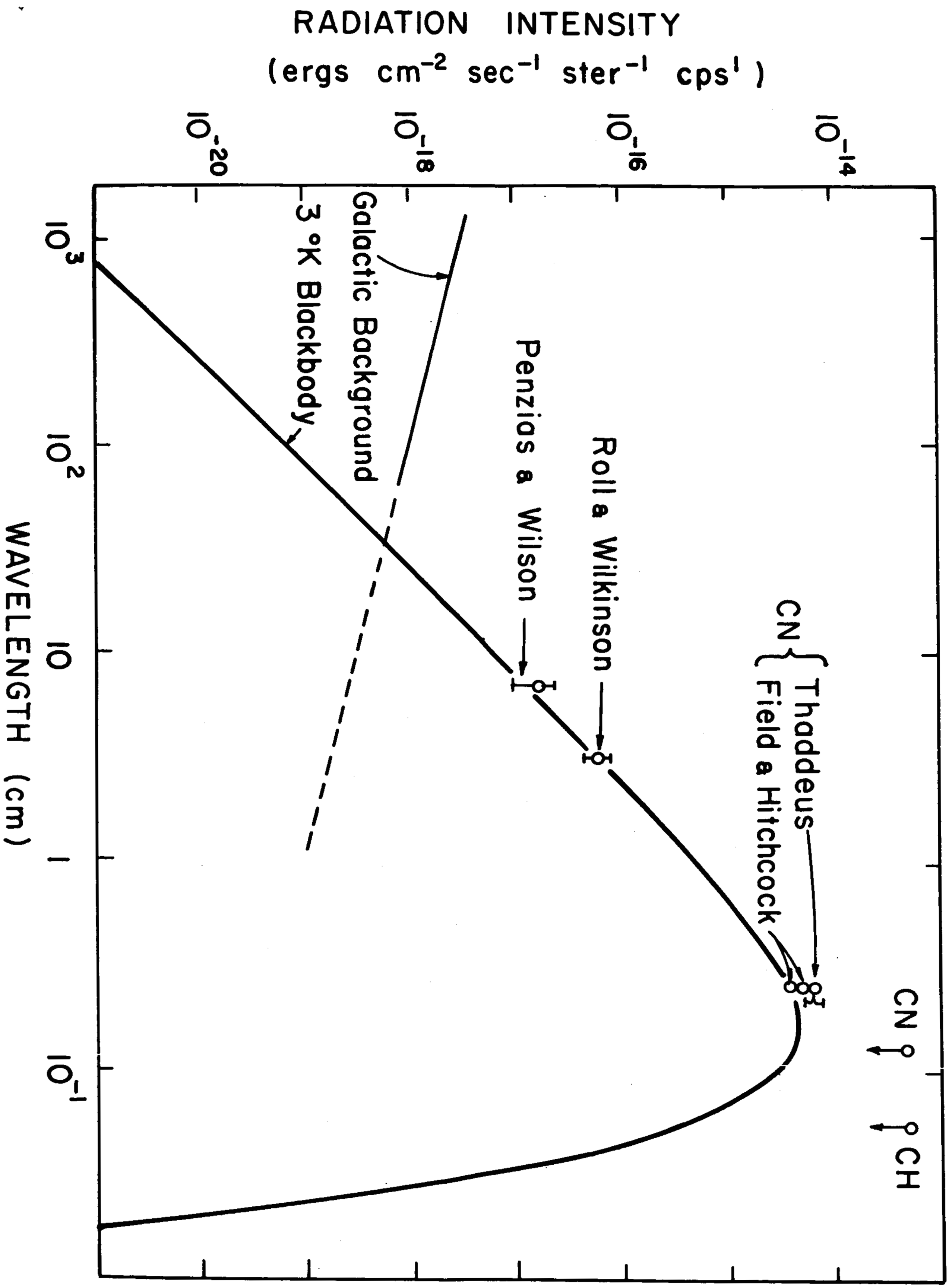

RADIATION INTENSITY
(ergs cm⁻² sec⁻¹ ster⁻¹ cps¹)
10⁻²⁰
10⁻¹⁸
10⁻¹⁶
10⁻¹⁴
Galactic Background
3 °K Blackbody
Penzias & Wilson
Roll & Wilkinson
CN Thaddeus
CN Field & Hitchcock
CN
CH
10³
10²
10
1
10⁻¹
WAVELENGTH (cm)

Fig. 1

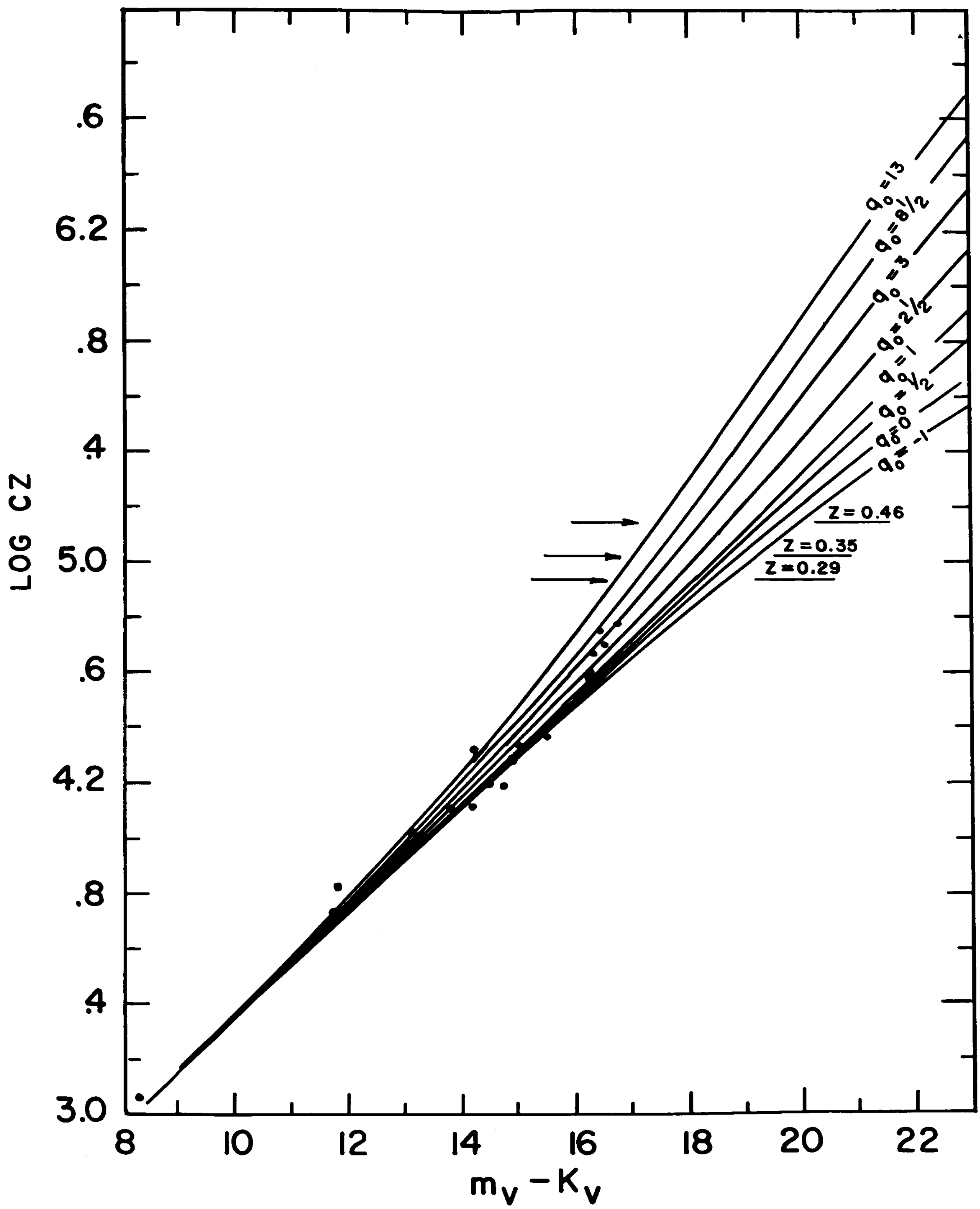
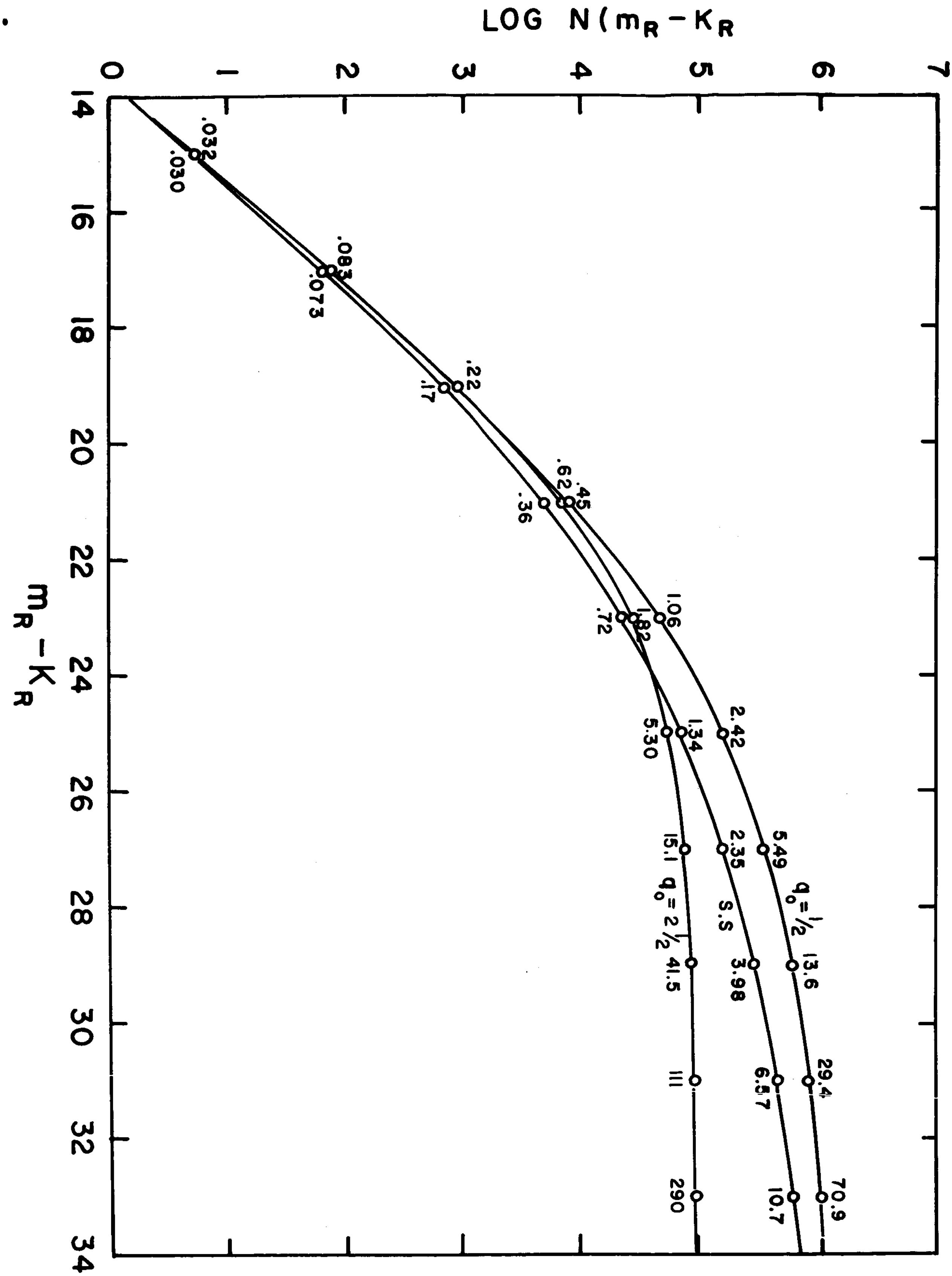

LOG CZ
.6
6.2
.8
.4
5.0
.6
4.2
.8
.4
3.0
8
10
12
14
16
18
20
22
$m_V - K_V$
$q_0=13$
$q_0=8\frac{1}{2}$
$q_0=3$
$q_0=2\frac{1}{2}$
$q_0=1$
$q_0=\frac{1}{2}$
$q_0=0$
$q_0=-1$
Z = 0.46
Z = 0.35
Z = 0.29

Fig. 2



LOG N(m_R - K_R
m_R - K_R
0
1
2
3
4
5
6
7
14
16
18
20
22
24
26
28
30
32
34
.032
.030
.083
.073
.22
.17
.62
.45
.36
1.06
1.82
.72
2.42
1.34
5.30
5.49
2.35
15.1
q_0 = 1/2
S.S
q_0 = 2 1/2
13.6
3.98
41.5
29.4
6.57
111
70.9
10.7
290

Fig . 3

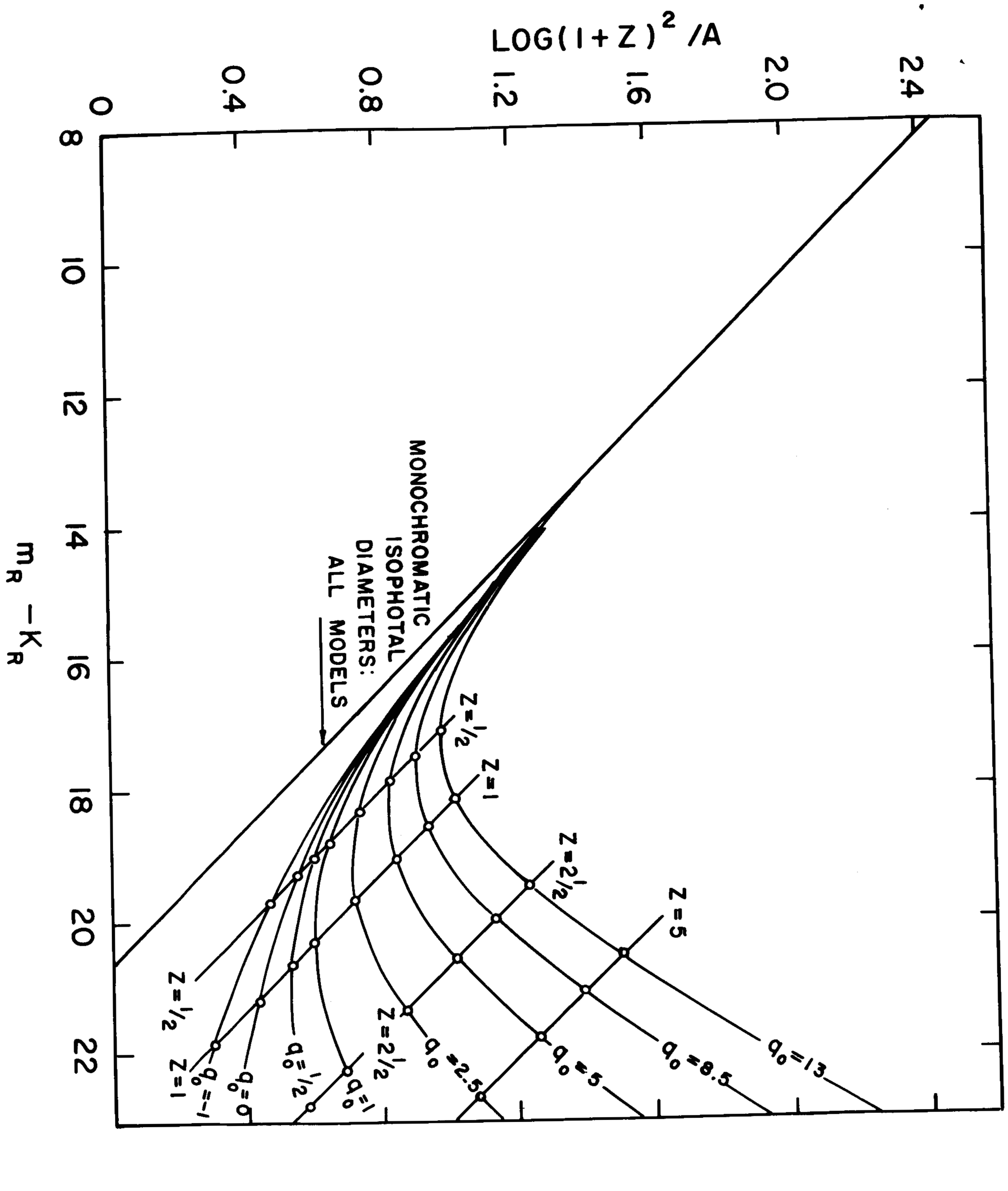

LOG(1+Z)^2/A
0
0.4
0.8
1.2
1.6
2.0
2.4
8
10
12
14
16
18
20
22
$m_R - K_R$
MONOCHROMATIC
ISOPHOTAL
DIAMETERS:
ALL MODELS
Z = 1/2
Z = 1
Z = 2 1/2
Z = 5
q_0 = 13
q_0 = 8.5
q_0 = 5
q_0 = 2.5
q_0 = 1
q_0 = 1/2
q_0 = 0
q_0 = -1

Fig. 4